Green Oaks Primary Academy
Calculation Policy

Without mathematics there is nothing you can do. Everything around you is mathematics; everything around you is numbers.

Shakuntala Devi

At Green Oaks Primary Academy, we wish to teach calculation with understanding, and not just as a process that is to be remembered. This Calculation Policy clarifies progression in calculation with examples that are 'mathematically transparent', in other words the way the calculation works is clear and supports both the development of mathematical concepts and closely links it to the mental strategies that are taught alongside the written methods.

AIMS OF THE POLICY

- To ensure consistency and progression in our approach to calculation and enable a smooth transition between year groups and phases.

- To ensure that children develop an efficient, reliable, formal written method of calculation for all operations.

- To ensure that children can use these methods accurately with confidence and understanding.

- To ensure pupils understand important concepts and make connections within mathematics.

- To ensure pupils show high levels of fluency in performing written and mental calculations.

- To ensure that pupils are ready for the next stage of learning and have been given strong foundations in mental methods, the use of practical equipment, allowed to explore jottings in a range of forms and then to move onto more formal recording using a strong knowledge of place value, number lines labelled or blank, partitioning before eventually using compact written methods.

- To ensure that pupils are competent in fluency, reasoning and problem solving and can make informed and appropriate choices about the methods they wish to use (mental or written) to solve mathematical problems efficiently and effectively.

Our calculation policy is broken down into sections. Each section is broken down into year group and highlights the methods pupils can expect to be taught, using a variety of manipulatives. Our policy provides explanations of the benefits of using the models shown and shows links between different operation. Following on from the methods, images then demonstrate the skill with model representations to effectively teach the concepts.

Our Calculation Policy incorporates a number of methods, most from Planpanion as this is the scheme, that we, as a school predominantly follow. However, to give pupils depth, breadth and experience we supplement our maths curriculum with a variety of resources and activities.

The basis of our maths calculation policy is that mental and written methods are integral to each other and should not be seen as taking separate paths but developed in conjunction with each other. It is envisaged that the development of mental skills will lead to jottings, (which support mental calculation) and then into more formalised jottings in the form of number lines and partitioning which in turn leads to expanded column methods and ultimately compact algorithms. ***It is important to always show the links between operations and not teach them in isolation or without showing, in practical problem-solving activities and across all mathematical topics, how these operations can be applied. It is important that staff always use correct mathematical language and encourage this from every pupil*** . This will take place in class discussions as well as through oral and written feedback, next steps and target setting.

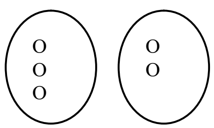
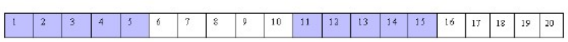
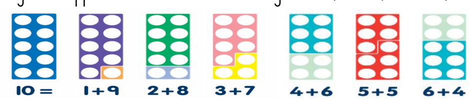
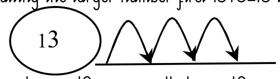
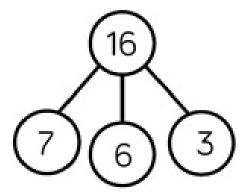
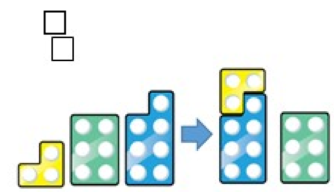
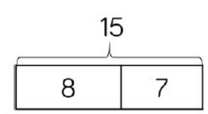
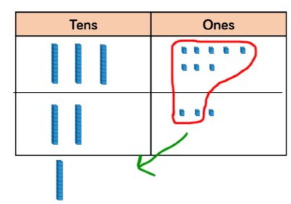
The statutory requirements for number, place value and fractions for each year group are detailed in Appendix 1.

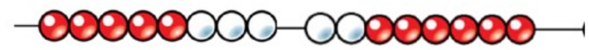
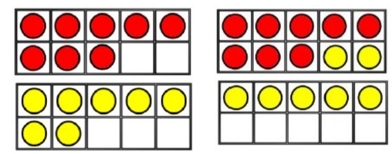
From Year 4 onwards children should be equipped with, and are able to choose from, the most appropriate of a range of mental, informal written and formal written methods. Although this policy exemplifies written procedures, the ability to calculate mentally lies at the heart of mathematics: it should be taught systematically and regularly as it is the essential basis of the subject.

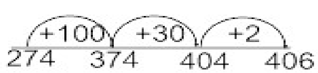
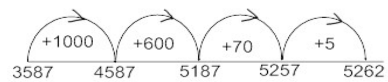
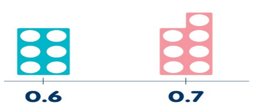
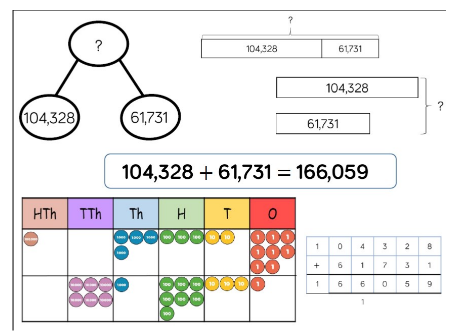
Fluency requires a bank of key facts which the children can recall at speed. These must be taught and practised regularly so that more complex aspects of computation are not halted by poor factual knowledge. **The facts to be learnt off by heart as stated in the revised curriculum are listed in Appendix 2.**

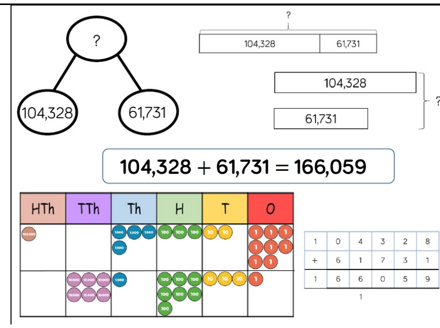
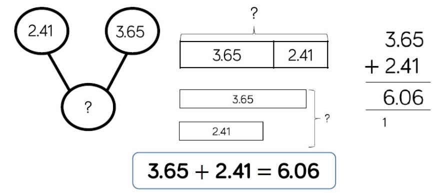
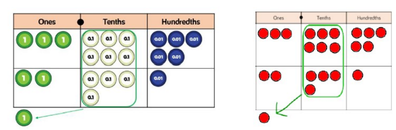
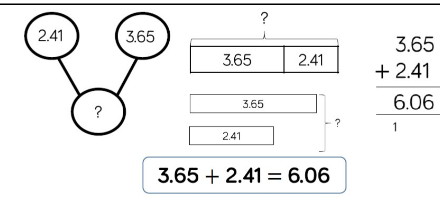
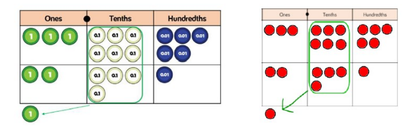
Our aim is that children leave our academy with a deep understanding of place value and how to manipulate number using the four rules. Every child should have a secure, competent and confident method for each of these rules. **Appendix 3 sets out examples of the formal written methods from the National Curriculum 2013.**

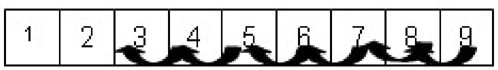
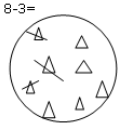
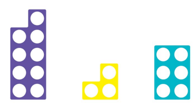
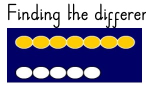
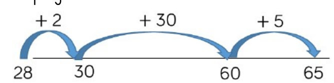
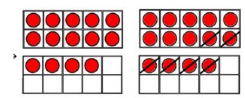
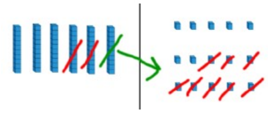
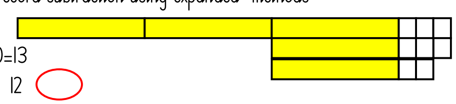
The national curriculum 2013 states that ***calculators*** should not be used as a substitute for good written and mental arithmetic, and therefore recommends they should only be introduced near the end of Key Stage 2 to support pupils' conceptual understanding and exploration of more complex number problems, if written and mental arithmetic are secure.

Addition		
Foundation	Year 1	Year 2
-Say the number that is one more than a number from 1 to 20. -Find the total number of items in two groups by counting all of them. -In practical activities and discussion, beginning to use the vocabulary involved in adding when combining two groups. -Count on and back from a number other than 0.	-Read, write and interpret mathematical statements involving addition (+) and (=) signs. -Represent and use number bonds within 20 -Add and one-digit and two-digit numbers to 20, including zero -Solve one-step problems that involve addition using concrete objects and pictorial representations, and missing number problems such as 4 + □ = 7	-Solve addition problems using concrete objects and pictorial representations, including: • a two-digit number and ones • a two-digit number and tens • two two-digit numbers • adding three one-digit numbers -Recall and use addition facts to 20 fluently, and derive and use related facts up to 100 -Show that addition of two numbers can be done in any order (commutative) -Use the inverse relationship between addition and subtraction to check calculations and solve missing number problems. -Start to record addition in columns.
Add, more, make, sum, total, altogether, one more, two more, ten more, how many more to make..?, how many more is…than…? Oral and practical work Songs and rhymes Dice and number games Number stories for combining sets eg 3 pigs in a field, 2 in a sty how many altogether? Teacher models 3+2=5 using a range of objects  Number track  Number bonds for numbers up to 10 Full number lines Tens frames	Songs and rhymes Working with apparatus such as bead strings to 20, cubes, Base 10, Numicon:  Use + and = signs and associated vocabulary. Adding more than 2 numbers Putting the larger number first 13+3=13 in your head or on fingers  Counting in 10s from multiples of 10 Number bonds of all numbers to 20 □+7=12 9+□ =18	Counting in 10s from any number Rapid recall of all number bonds for all numbers to 20. Use of Numicon, Base 10, bar model, part whole model to demonstrate.   7 + 6 + 3 = 16   Structured number lines and bead strings to 100

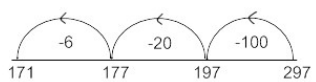
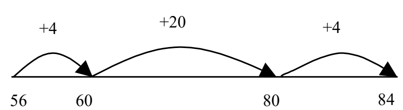
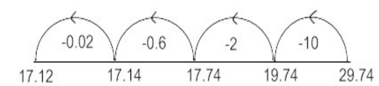
		 Use of tens frames  8 + 7 = 15 2 5 Start to record addition in columns, using expanded methods 40+ 7 30+ 6 70+13 = 83 Can check by using inverse operation, use to solve missing box problems Eg $\boxed{}$ + 5= 23

Addition			
Year 3	Year 4	Year 5	Year 6
-Add a range of numbers mentally, including: a three-digit number and ones a three-digit number and tens a three-digit number and hundreds -Add numbers with up to three digits, using formal written methods of columnar addition -Estimate the answer to a calculation and use inverse operations to check answers -Solve problems, including missing number problems, using number facts, place value, and more complex addition. -Add fractions with the same denominator within one whole (for example, $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$)	-Add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate -Estimate and use inverse operations to check answers to a calculation -Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why. -Add fractions with the same denominator -Solve simple measure and money problems involving fractions and decimals to two decimal places	-Add whole numbers with more than 4 digits, including using formal written methods (columnar addition) -Add numbers mentally with increasingly large numbers (eg. 8 462 + 2300 = 10 762). -Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy -Solve addition and subtraction multi-step problems in contexts, including to 3 decimal places, deciding which operations and methods to use and why. -Add and subtract fractions with the same denominator and denominators that are multiples of the same number	-Add whole numbers with more than 4 digits, including using formal written methods (columnar addition) -Perform mental calculations, including with mixed operations and large numbers -Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why -Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
Use partitioning to support mental calculations. Using an empty number line to count on. 274 + 132  Add a near multiple of 10 to a two-digit number Continue as in Year 2 but with appropriate numbers e.g. 350 + 189 is the same as 350 + 190 – 1. Extend use of columnar addition, developing more compact recording to tackle larger numbers. 40+ 7 30+ 6 70+13 = 83	Using an empty number line to count on. 3587+1675  Extend use of expanded columnar addition to 4 digit numbers, leading to the use of the compact method. Extend to decimals, assigning values to Numicon and bar models to support.  	Use formal columnar addition for numbers with more than 4 digits. 21271 12243 + 33514 1 Including method were carrying is used. Extend to decimals. 42.432 12.713 + 55.145 1 Develop reasoning skills by using a range of representations including part whole models, number sentences, place value counter problems and bar models.	Use formal column addition for any numbers which cannot be added mentally (>1 million) 2353248 1254173 + 3607421 1 11 

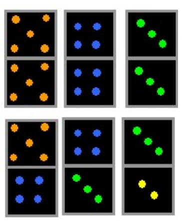
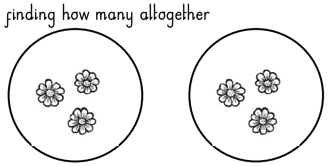
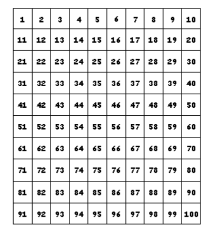
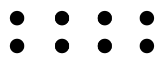
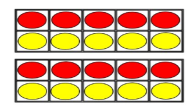
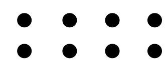
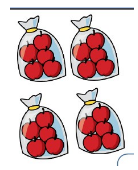
		   Add fractions with the same denominator and multiples of the same number. $\frac{2}{3} + \frac{1}{6} = \frac{4}{6} + \frac{1}{6} = \frac{5}{6}$ Solve problems involving all the above.	  Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions $\frac{1}{3} + \frac{1}{5} = \frac{5}{15} + \frac{3}{15} = \frac{8}{15}$

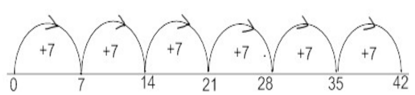
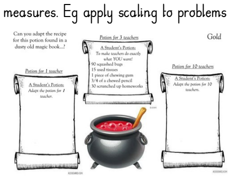
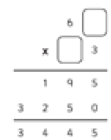
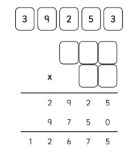
Subtraction		
Foundation	**Year 1**	**Year 2**
-Say the number that is one less than a number from 1 to 20. -In practical activities and discussion, beginning to use the vocabulary involved in subtraction when taking away objects groups.	-Read, write and interpret mathematical statements involving addition (-) and (=) signs. -Represent and use number bonds and related subtraction facts within 20 -Subtract one-digit and two-digit numbers to 20, including zero -Solve one-step problems that involve subtraction using concrete objects and pictorial representations, and missing number problems such as 8 - □ = 5	-Solve subtraction problems using concrete objects and pictorial representations, including: • a two-digit number and ones • a two-digit number and tens • two two-digit numbers -Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100 -Show that subtraction of one number from another cannot be done in any order. -Use the inverse relationship between addition and subtraction to check calculations and solve missing number problems. -Start to record subtraction in columns.
Take (away), leave, how many are left/left over?, How many have gone? One less, two less, ten less, how many fewer is..? difference between, is the same as Oral and practical work Songs and rhymes Dice and number games, counting back, taking away. Use of number tracks.  Number stories using objects 8-3=  How many are there? How many now? (after some have been removed) Teacher modelling number sentences, 8 take away 3 is 5	Songs and rhymes Working with apparatus Bead strings to 20. Cubes, Base 10, bar model.  Subtraction with Numicon.  9 - 3 = 6 Physical and practical work on structured number lines eg jumping backwards Number stories, 15 people on a bus 3 get off, how many are left on? Putting a number in your head and counting back with fingers to help. 12-3= □ 15- □ = 4 □ -11=4 Counting back in 10s from multiples of 10s Giving change to 20p Finding the difference by counting on, comparing quantities 	Counting back in 10s from any number to 100 Jumping back on a structured number line. 76-34    Finding the difference between 2 towers of cubes leading to using the structured number line or fingers for numbers that are close together to calculate difference by counting on eg 42-39= 3 Use addition as the inverse operation to check and in empty box problems eg □ - 8=12 Start to record subtraction using expanded methods 33 – 21 33 – 20=13 13 – 1 = 12 

Subtraction

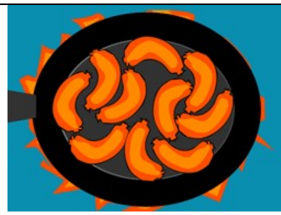
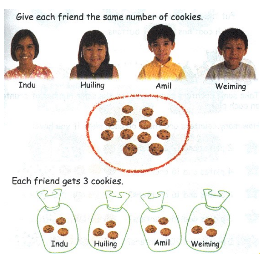
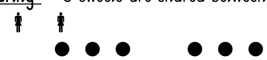
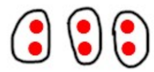
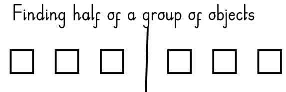
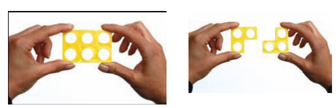
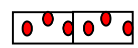
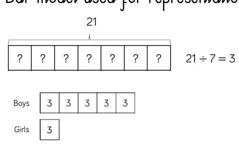
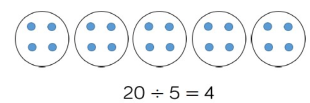
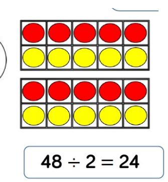
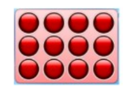
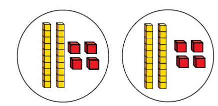
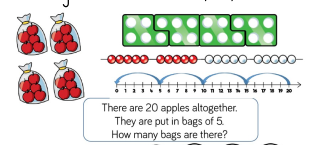
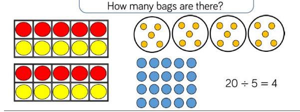
Year 3	Year 4	Year 5	Year 6
-Subtract a range of numbers mentally, including: • a three-digit number and ones • three-digit number and tens • a three-digit number and hundreds -Subtract numbers with up to three digits, using formal written methods of columnar subtraction -Estimate the answer to a calculation and use inverse operations to check answers -Solve problems, including missing number problems, using number facts, place value, and more complex addition. -Subtract fractions with the same denominator within one whole (for example, $^5/_7 - {}^1/_7 = {}^4/_7$)	-Subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate -Estimate and use inverse operations to check answers to a calculation -Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why. -Subtract fractions with the same denominator -Solve simple measure and money problems involving fractions and decimals to two decimal places	-Subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction) -Subtract numbers mentally with increasingly large numbers (eg. 10 462 - 2300 = 8 162). -Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy -Solve addition and subtraction multi-step problems in contexts, including to 3 decimal places, deciding which operations and methods to use and why. -Add and subtract fractions with the same denominator and denominators that are multiples of the same number	-Subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction) -Perform mental calculations, including with mixed operations and large numbers -Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why -Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
Develop confidence in counting back in 100s, 10s and 1s from any number. Use an empty number line to count back. 297-126=  Count on to find the difference using empty numberline 84-56=  Expanded method of decomposition for numbers too large to do mentally. 572-158= 60 12 500 ~~70~~ ~~2~~ 100 50 8 – 400 10 4 = 414	Using an empty number line to both count back, and find the difference between two numbers by counting on. (Up to 4 digits) Extend to decimals to 2 decimal places.  Support by re-assigning values to Numicon.  Expanded method of decomposition, leading to more compact recording. 2757-1259= 6 14 17 2 7 ~~5~~ ~~7~~ 1 2 5 9 - 1 4 9 8 Extend to decimals.	Formal method used for both calculations with and without borrowing 874 – 523 becomes 8 7 4 – 5 2 3 3 5 1 Answer: 351 932 – 457 becomes 8 12 1 ~~9~~ ~~3~~ 2 – 4 5 7 4 7 5 Answer: 475 Move towards compact decomposition, including decimals. 2 1 ~~3~~6.57 17.46 – 19.11 Subtract fractions with the same denominator and multiples of the same number. $^2/_3 - {}^1/_6 = {}^4/_6 - {}^1/_6 = {}^3/_6$	Use formal method of compact decomposition. 2 1 ~~3~~6.573 18.462 – 18.111 Apply to problem solving contexts eg money and measures Subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions $^1/_3 - {}^1/_5 = {}^5/_{15} - {}^3/_{15} = {}^2/_{15}$ Revert to expanded methods if the children experience any difficulty.

Multiplication

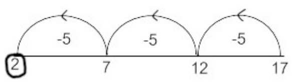
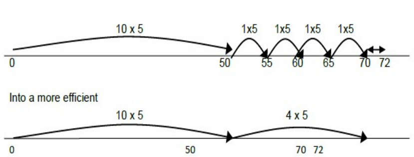
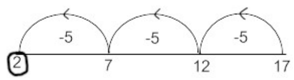
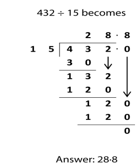
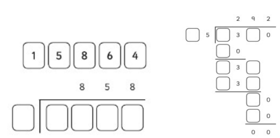
Foundation	Year 1	Year 2
-Start to solve problems involving doubling.	-Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher. -Make connections between arrays, number patterns, and counting in twos, fives and tens.	-Recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers -Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs -Show that multiplication of two numbers can be done in any order (commutative) -Solve problems involving multiplication using materials, arrays, repeated addition, mental methods and multiplication and including problems in contexts.
Counting in ones, twos, tens Odd and even numbers Matching pairs eg socks Noahs ark    Songs and rhymes Finding doubles in dominoes Doubles in practical contexts. Groups of objects with the same number, counting how many in each group, and finding how many altogether 	Counting in twos, fives and tens (using fingers to help count in multiples) Knowing doubles of numbers to 10 Dice and domino games with doubles Finding patterns of numbers using a 100 square and make connections with arrays.   Repeated addition of sets of objects, teacher modelling 2+2+2 = 6 Use coins for repeated addition  and model using Numicon.  5 + 5 + 5 = 15	Counting in 3s Doubles of all numbers up to 10 and doubles of multiples of 10 to 100 Recognise odd and even numbers, supported by Tens frames.  Arrays and repeated addition this links to commutative law below. Use visual and concrete methods below as long with fingers for counting. Additional language introduced including "lots of" for problem solving.  4 x 2 or 4 + 4 2 x 4 or 2 + 2 + 2 + 2  Commutative law 4 x 3 = 12 3 X 4 = 12

Multiplication			
Year 3	Year 4	Year 5	Year 6
-Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables -Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods of short multiplication -Solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.	-Recall multiplication and division facts for multiplication tables up to 12 × 12 -Use place value, known and derived facts to multiply mentally, including multiplying by 0 and 1 and multiplying together three numbers -Recognise and use factor pairs and commutativity in mental calculations -Multiply two-digit and three-digit numbers by a one-digit number using formal written layout -Solve problems involving multiplying including using the distributive law to multiply two-digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.	-Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers -Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers -Multiply numbers mentally drawing upon known facts, including multiplying whole numbers and those involving decimals by 10, 100 and 1000 -Recognise and use square numbers and cube numbers, write the notation for both [(^{2}) and (^{3})] and solve problems involving multiplication using knowledge of factors and multiples, squares and cubes -Solve problems involving scaling by simple fractions. -Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams	-Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication -Multiply one-digit numbers with up to two decimal places by whole numbers -Perform mental calculations, including with mixed operations and large numbers -Multiply simple pairs of proper fractions -Identify common factors, common multiples and prime numbers
Doubling multiples of 5 up to 50 by partitioning 15 x 2 = 30 10 + 5 20 + 10 = 30 Know that division is inverse of multiplication and multiplication is inverse of division Understand multiplication as repeated addition Use a number line to solve 6x7  Continue to use arrays Use expanded column method as a step towards short multiplication. 24 x 6 = 20 4 X 6 120 + 24 = 144 Progress towards formal short multiplication 24 × 6 becomes 2 4 × 6 1 4 4 2	Multiplication by using known facts. Eg to multiply by 60, multiply by 6 then X10 Doubling all numbers to 50, multiples of 10 to 500 Multiply decimals and integers by 10, 100 and1000. Use the grid method 234 x 7= Extend use of formal short multiplication 342 × 7 becomes 3 4 2 × 7 2 3 9 4 2 1 Missing number type problems e.g. 12 x ? = 9 x 8. Use methods within problem solving contexts such as money and measures. Eg apply scaling to problems such as recipes and ingredients. 	Use short multiplication when multiplying by 1 digit. 342 × 7 becomes 3 4 2 × 7 2 3 9 4 2 1 Answer: 2394 Use formal long multiplication for up to 4 digit x2 digit 124 × 26 becomes 1 2 1 2 4 × 2 6 7 4 4 2 4 8 0 3 2 2 4 1 1 Answer: 3224  Multiply proper fractions, ½ x $^2/_5$ Missing number problems eg	Use formal long multiplication for up to 4 digits x 2 digits. Eg 1354 x 24 1 2 11 1354 X 24 5416 27080 32496 1 Extend to decimals. Multiply simply pairs of proper fractions. $\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$ Missing number problems Eg using the given digit cards once, complete the calculation 

Division

Foundation	Year 1	Year 2
-Start to solve problems involving halving and sharing	-Solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	-Recall and use division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers -Calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs -Show that division of one number by another cannot be done in any order -Solve problems involving division, using materials, arrays, repeated subtraction, mental methods, and multiplication and division facts, including problems in contexts.
Practical activities, songs and rhymes. 10 fat sausages…  Sharing during snack time by giving 1 each Is there an easier way of sharing a larger amount? Eg 2 at a time  Making groups/piles of 2, finding partners Eg in PE grouping in 2s, how many pairs are there? 1 ball for each pair, how many balls do I need to get out?	Practical activities, songs, and games. Sharing – 6 sweets are shared between 2 people. How many do they have each?  Grouping – There are 6 sweets. How many people can have 2 each? (How many 2's make 6?)  Cutting cakes/ pizza in half, sharing related to fractions Finding half of a group of objects  Knowing halves of even numbers to 20  Use Numicon and bar model as a representation   Bar model used for representation of groups in a whole  	Relate division to fractions ½ or ¼ of 12, 20. Half of 12 is 12÷2=. Introduction of fact families and making equal groups Understand division as sharing and grouping and link to multiplication facts 12÷ 3 = 4 12÷ 4 = 3 3 X 4 = 12 4 X 3 = 12     Counting on and back in 2s, 5s, 10s. How many 2s in 10?   Counting on fingers also used at this stage. ÷ = signs and missing numbers 6 ÷ 2 = □ □ = 6 ÷ 2 6 ÷ □ = 3 3 = 6 ÷ □ □ ÷ 2 = 3 3 = □ ÷ 2

Division

Year 3	Year 4	Year 5	Year 6
-Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables -Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, using mental and progressing to formal written methods -Solve problems, including missing number problems, involving multiplication and division -Calculate simple remainders after division	-Recall multiplication and division facts for multiplication tables up to 12 × 12 -Recognise and use factor pairs in mental calculations -Divide two-digit and three-digit numbers by a one-digit number using formal written layout -Divide a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths	-Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context -Divide whole numbers and those involving decimals by 10, 100 and 1000 -Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers -Establish whether a number up to 100 is prime and recall prime numbers up to 19	-Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context -Identify common factors, common multiples and prime numbers -Divide proper fractions by whole numbers (for example, $\frac{1}{3} \div 2 = \frac{1}{6}$) -Associate a fraction with division and calculate decimal fraction equivalents (for example, 0.375) for a simple fraction (for example, $\frac{3}{8}$)
¼ or ½ of 24 , 40.. etc Understand division as grouping and as sharing. *e.g. If there are 14 sweets in a bag, how many people can have 2 each?* Practically demonstrate repeated subtraction to find how many groups. Remainders 17÷5= 3r2  Make clear links between x and ÷ ÷ = signs and missing numbers Divide by 10 and 100 Extend to pencil and paper procedures which reflect mental methods. The number line is also an excellent way of introducing the 'chunking' approach. 72 ÷ 5 = 14 r 2  	Sharing and grouping Continue to understand division as both sharing and grouping (repeated subtraction). Remainders 17÷5= 3r2  Approximate first. Use informal or pictorial methods relating to the child's mental methods moving onto short formal method when ready. 98 ÷ 7 becomes 1 4 7 | 9 ²8	Consolidate formal short division 432 ÷ 5 becomes 8 6 r 2 5 | 4 3 ³2 Complete missing number calculations 6 5 9 | □ 8 □ Quotients can be expressed as fractions or decimal fractions 61 ÷ 4 = 15 ¼ or 15.25	Formal short division for 4 digit ÷ 1 digit 1 8 6. 2 5 | 9 ⁴3 ³1. ¹0 (remainders shown as a decimal) This method can also be used for decimals. Introduce formal long division  Division of fractions using **keep, change, flip.** $\frac{3}{5} \div \frac{2}{8} = \frac{3}{5} \times \frac{8}{2} = \frac{24}{10} = 2\frac{4}{10}$ $\frac{3}{5} \div 2 = \frac{3}{5} \div \frac{2}{1} = \frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$ $2\frac{3}{4} \div \frac{1}{4} = \frac{11}{4} \div \frac{1}{4} = \frac{11}{4} \times \frac{4}{1} = \frac{44}{4} = 11$ Missing number questions eg Using the digit cards given, complete the calculation  Leading to long division with missing numbers.

Appendix 1: Place Value (including fractions)

Number, place value and fractions statutory requirements (National Curriculum 2013, Early Years Foundation Stage 2021).

Early Years Foundation Stage

Pupils should be taught to:

- Number rhymes/ stories/ songs – Action rhymes.
- Sorting into sets.
- Matching:
    - object - object
    - object – picture
    - picture – picture
    - patterns
- **Comparing**
    - Arranging/ organise items into sets.
    - Sorting – 1 general set.
    - Sorting for positive/ negative reasons.
    - Sorting for specific reasons.
    - Sorting for specific reasons chosen by the child.
- **Order**
    - Objects which are bigger/ smaller.
    - Ordering numerals on a washing line/ number line/magnetic numbers.
- **Sequencing**
- - Recognising patterns.
- - Copying a pattern.
- - Creating a pattern.
- **Count**
- - Learn number words.

- - Recognising some numerals.
- - Counting how many?

- **Recognising numbers**

- - Number formation practise.
- - Flash cards.
- - Number snap/ puzzles/ games.
- - Magnetic numbers.
- - Printing numbers.
- - Painting numbers.
- - Playdough numbers.
- - Big Book work.
- - Display work.
- - IT programs.

Year 1

- Pupils should be taught to:
- count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number
- count, read and write numbers to 100 in numerals, count in multiples of twos, fives and tens
- given a number, identify one more and one less
- identify and represent numbers using objects and pictorial representations including the number line, and use the language of equal to, more than, less than (fewer), most, least
- read and write numbers from 1 to 20 in numerals and words
- read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = – 9.
- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- recognise, find and name a half as one of two equal parts of an object, shape or quantity
- recognise, find and name a quarter as one of four equal parts of an object, shape or quantity

Year 2

Pupils should be taught to:

- count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward
- recognise the place value of each digit in a two-digit number (tens, ones)
- identify, represent and estimate numbers using different representations, including the number line
- compare and order numbers from 0 up to 100; use <, > and = signs
- read and write numbers to at least 100 in numerals and in words
- use place value and number facts to solve problems
- solve problems with addition and subtraction:
- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods
- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems
- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in context
- recognise, find, name and write fractions $^1/_3$, $^1/_4$, $^2/_4$ and $^3/_4$ of a length, shape, set of objects or quantity
- write simple fractions for example, $^1/_2$ of $6 = 3$ and recognise the equivalence of $^2/_4$ and $^1/_2$

Year 3

Pupils should be taught to:

- count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number
- recognise the place value of each digit in a three-digit number (hundreds, tens, ones)
- compare and order numbers up to 1000
- identify, represent and estimate numbers using different representations
- read and write numbers up to 1000 in numerals and in words
- solve number problems and practical problems involving these ideas
- add and subtract numbers mentally, including:
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds
- add and subtract numbers with up to three digits, using formal written methods of column addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.
- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects
- count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10
- recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators
- recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators
- recognise and show, using diagrams, equivalent fractions with small denominators
- add and subtract fractions with the same denominator within one whole (for example, $^5/_7 + {}^1/_7 = {}^6/_7$)
- compare and order unit fractions, and fractions with the same denominators
- solve problems that involve all the above

Year 4

Pupils should be taught to

- count in multiples of 6, 7, 9, 25 and 1000
- find 1000 more or less than a given number
- count backwards through zero to include negative numbers
- recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)
- order and compare numbers beyond 1000
- identify, represent and estimate numbers using different representations
- round any number to the nearest 10, 100 or 1000
- solve number and practical problems that involve all the above and with increasingly large positive numbers
- read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value
- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why
- recall multiplication and division facts for multiplication tables up to 12 × 12
- use place value, known and derived facts to multiply and divide mentally, including multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects
- recognise and show, using diagrams, families of common equivalent fractions
- count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten
- solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
- add and subtract fractions with the same denominator
- recognise and write decimal equivalents of any number of tenths or hundredths
- recognise and write decimal equivalents to $^1/_4$, ½ and ¾
- find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths
- round decimals with one decimal place to the nearest whole number
- compare numbers with the same number of decimal places up to two decimal places
- solve simple measure and money problems involving fractions and decimals to two decimal places

Year 5

Pupils should be taught to:

- read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit
- count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000
- interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero
- round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000
- solve number problems and practical problems that involve all of the above
- read Roman numerals to 1000 (M) and recognise years written in Roman numerals
- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (nonprime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)
- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.
- compare and order fractions whose denominators are all multiples of the same number
- identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths
- recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $^{2}/_{5} + {^{4}/_{5}} = {^{6}/_{5}} = 1\,{^{1}/_{5}}$]
- add and subtract fractions with the same denominator and denominators that are multiples of the same number
- multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
- read and write decimal numbers as fractions [for example, $0.71 = {^{71}/_{100}}$]
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents

- round decimals with two decimal places to the nearest whole number and to one decimal place
- read, write, order and compare numbers with up to three decimal places
- solve problems involving number up to three decimal places
- recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal
- solve problems which require knowing percentage and decimal equivalents of ½ , ¼ , $^1/_5$, $^2/_5$, $^4/_5$ and those fractions with a denominator of a multiple of 10 or 25

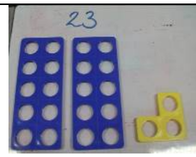
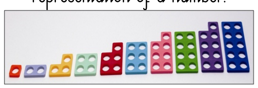
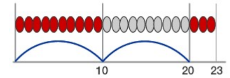
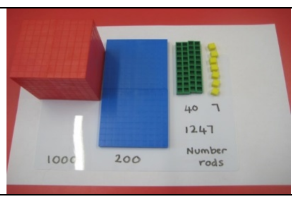
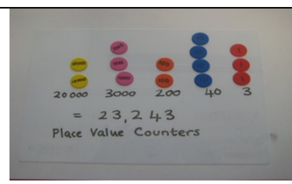
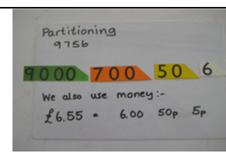
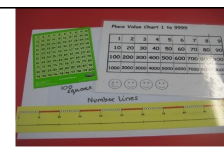
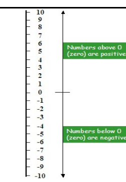
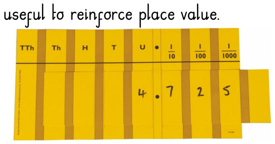
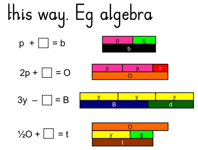
Year 6

Pupils should be taught to:

- read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- round any whole number to a required degree of accuracy
- use negative numbers in context, and calculate intervals across zero
- solve number and practical problems that involve all of the above
- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.
- use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- compare and order fractions, including fractions > 1
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- multiply simple pairs of proper fractions, writing the answer in its simplest form (for example, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$)
- divide proper fractions by whole numbers (for example, $\frac{1}{3} \div 2 = \frac{1}{6}$)
- associate a fraction with division and calculate decimal fraction equivalents (for example, 0.375) for a simple fraction (for example, $\frac{3}{8}$)
- identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places
- multiply one-digit numbers with up to two decimal places by whole numbers
- use written division methods in cases where the answer has up to two decimal places
- solve problems which require answers to be rounded to specified degrees of accuracy
- recall and use equivalences between simple fractions, decimals and percentages, including in different contexts

Representations for Place Value and Calculation

In line with the National Curriculum Aims manipulatives should be used to develop an understanding of the mathematical concept and to build a firm foundation in calculations. Some children will prefer some representations more than others and may not use all of them. They all will progress at different rates. Practical handling of resources is essential to aid secure understanding at all stages.

				
Initially objects, counters cubes etc are used to reinforce the link between counting, one to one correspondence, the verbal cardinal and written number. Numicon is introduced as a pictorial representation of a number. 	Once 1:1 correspondence is well established, children can recognise one item to represent a number other than one – such as Numicon or Cuisenaire. Children soon learn what number each colour represents and use these to make numbers. Numicon is very useful for seeing which numbers are odd, because they always have an extra piece sticking up whereas the even numbers are flat across both ends.	Numbers can be made out of straws and children learn to bundle the straws into piles of 10 with an elastic band to be able to make numbers quicker. They need to make lots of bundles of 10 before they know a bundle is always 10. 20 bead strings are also useful to see the groups of tens and individual beads. 	Once the place value of 10 is secure with straws, number rods can be used to make numbers, where the ten rod cannot be broken into ones. 100 bead strings are useful at this stage. 	Once children understand that a 10p coin is the same as ten 1p coins, and a £1 coin is 100 pennies, they can make amounts over £1. Money is often the easiest way to introduce decimals to children once they get into KS2.
				
Lots of practice using number rods and blocks help children appreciate the relevant size of each number in its place value position.	When children are very secure with using number blocks and rods they can progress onto using Place Value Counters. Each colour counter represents a place value and to help these are also written on each colour. They start by building numbers and exploring bigger numbers; how to write and say them.	Breaking the number into its parts or partitioning helps secure understanding of place value and is an important step in beginning calculations. Sliders for multiplying and dividing by 10 are very useful to reinforce place value. 	Hundred squares, number lines and place value charts to help children understand the value of the digit in each position. The versatility of a number line is that, although the divisions are usually initially numbered in ones, they can represent any concept such as counting in 10s, 100s, decimals or fractions.	Number lines can show the value of negative as well as positive numbers. Cuisenaire rods are very useful when introducing the bar model as a pictorial representation. These should be used whenever a new concept is represented in this way. Eg algebra 

Manipulatives are not to be used sequentially but as and when they are appropriate. All classrooms should have manipulatives for pupils to access. Encouragement should be given through the school to all pupils to promote and question *is this the most efficient method of working out this calculation?*

As a new concept is introduced the use of a previously abandoned representation may help clarify and aid understanding eg using straws for fractions, where the bundle represents 1 instead of 10 and therefore each straw represents 1/10th.

Appendix 2

The National Curriculum (DfE, 2013) expects the **number facts** that should be known by heart, by the end of each year are:

Year 1:

- Pairs of numbers that make numbers up to 10 and 20 in several forms
- *e.g. 3 + 4 = 7; 7 - 3 = 4; 3 = 7 - 4; 9 +7 = 16; 16 − 7 = 9; 7 = 16 − 9*
- Doubles of numbers up to 10 + 10 *e.g. double 1 = 2, double 2 = 4, double 3 = 6…*
- and their corresponding halves *e.g. half of 10 is 5, half of 12 is 6;*

Year 2:

- **All** addition and subtraction facts for each number to 20 *e.g. 9 +7 = 16; 16 − 7 = 9; 7 = 16 − 9*
- Use these facts to derive others to 100 *e.g. 3 + 6 = 9 so 30 + 60 = 90*
- Multiplication facts for the 2, 5 and 10 times tables *e.g. 2 x 4 = 8, 5 x 6 = 30, 10 x 7 = 70;*
- Division facts for the 2, 5 and 10 times tables
- *e.g. I know that 3 x 5 = 15, so I also know that 15 ÷ 5 = 3*
- Recognise odd and even numbers

Year 3:

- Multiplication facts for the 3, 4 and 8 times tables;
- Derive division facts from these known multiplication facts
- *e.g. I know that 6 x 8 = 48 therefore 48 ÷ 6 = 8*
-

Year 4:

- Multiplication facts for the 6, 7, 9, 11 and 12 times tables;
- Derive division facts from these known multiplication facts
- *e.g. I know that 7 x 9 = 63 therefore 63 ÷ 9 = 7*
-

Year 5:

- Square numbers up to 10 x 10 *e.g. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100;*
- Cubed numbers *e.g. 1x1x1 = 1, 2x2x2 = 8, 3x3x3 = 27, 4x4x4 = 64, 5x5x5 = 125*
- Prime numbers up to 100*e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37…*

Appendix 3.

Examples of formal written methods for addition, subtraction, multiplication and division (National Curriculum 2013).

Addition and Subtraction

789 + 642 becomes

	7	8	9
+	6	4	2
1	**4**	**3**	**1**
	1	1	

Answer: 1431

874 – 523 becomes

	8	7	4
–	5	2	3
	3	**5**	**1**

Answer: 351

932 – 457 becomes

	8	12	1
	$\cancel{9}$	$\cancel{3}$	2
–	4	5	7
	4	**7**	**5**

Answer: 475

932 – 457 becomes

		1	1
	9	3	2
–	$\cancel{4}$	$\cancel{5}$	7
	5	6	
	4	**7**	**5**

Answer: 475

Short Multiplication

24 × 6 becomes

	2	4
×		6
1	**4**	**4**
	2	

Answer: 144

342 × 7 becomes

	3	4	2
×			7
2	**3**	**9**	**4**
	2	1	

Answer: 2394

2741 × 6 becomes

	2	7	4	1
×				6
1	**6**	**4**	**4**	**6**
	4	2		

Answer: 16 446

Long Multiplication

24 × 16 becomes

$$\begin{array}{rrr} & {}^{2}2 & 4 \\ \times & 1 & 6 \\ \hline 2 & 4 & 0 \\ 1 & 4 & 4 \\ \hline 3 & 8 & 4 \\ \hline \end{array}$$

Answer: 384

124 × 26 becomes

$$\begin{array}{rrrr} & {}^{1}1 & {}^{2}2 & 4 \\ \times & & 2 & 6 \\ \hline 2 & 4 & 8 & 0 \\ & 7 & 4 & 4 \\ \hline 3 & 2 & 2 & 4 \\ \hline {}_{1} & {}_{1} & & \end{array}$$

Answer: 3224

124 × 26 becomes

$$\begin{array}{rrrr} & {}^{1}1 & {}^{2}2 & 4 \\ \times & & 2 & 6 \\ \hline & 7 & 4 & 4 \\ 2 & 4 & 8 & 0 \\ \hline 3 & 2 & 2 & 4 \\ \hline {}_{1} & {}_{1} & & \end{array}$$

Answer: 3224

Short Division

98 ÷ 7 becomes

$$\begin{array}{r|rr} & 1 & 4 \\ \hline 7 & 9 & {}^{2}8 \end{array}$$

Answer: 14

432 ÷ 5 becomes

$$\begin{array}{r|rrrl} & & 8 & 6 & r\,2 \\ \hline 5 & 4 & 3 & {}^{3}2 & \end{array}$$

Answer: 86 remainder 2

496 ÷ 11 becomes

$$\begin{array}{r|rrrl} & & 4 & 5 & r\,1 \\ \hline 1\;1 & 4 & 9 & {}^{5}6 & \end{array}$$

Answer: $45\frac{1}{11}$

Long Division

432 ÷ 15 becomes

			2	8	r 12
1	5	4	3	2	
		3	0	0	
		1	3	2	
		1	2	0	
			1	2	

Answer: 28 remainder 12

432 ÷ 15 becomes

			2	8	
1	5	4	3	2	
		3	0	0	15×20
		1	3	2	
		1	2	0	15×8
			1	2	

$\frac{\cancel{12}}{\cancel{15}} = \frac{4}{5}$

Answer: 28 $\frac{4}{5}$

432 ÷ 15 becomes

			2	8·	8
1	5	4	3	2·	0
		3	0	↓	↓
		1	3	2	
		1	2	0	
			1	2	0
			1	2	0
					0

Answer: 28·8

As a school, we do not dictate which method is the preferred and a range of methods can be taught enabling the pupils to choose the method most efficient to them in their own workings.